The Impact of Atmospheric Turbulence on Telescope Images, as a Function of Wavelength: Comparison of Theory and Simulations

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Abstract. The next generation of ground-based surveys such as the Large Synoptic Survey Telescope (LSST) will offer sufficient data volume to reduce statistical uncertainties for weak gravitational lensing – which will offer insights into cosmic expansion driven by dark energy.[4] Yet weak lensing inference works on telescope images containing distortions due to the atmosphere, and the increased data volume may amplify the effect of systematic bias. One source of such bias is that atmospheric turbulence blurs images in a wavelength-dependent fashion. The degree of the blurring is approximated by the width of the atmospheric point spread function (PSF), which governs how a point source of light is altered once it passes through the atmosphere. To that end, this paper establishes predictions made by the von Kármán model as to how the width of the atmospheric PSF varies as a function of source wavelength – then compares the relationship with that in simulations from GalSim, a galaxy simulation package. We find agreement at maximum turbulence scales 75m, 100m, 1000m, and infinity and disagreement at 25m and 50m. Given that 20m is the measured value of the outer scale at the LSST site, we recommend matching more parameters between theory and simulations, such as telescope exposure time, and confirming whether the disagreement persists.
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1 Introduction

The Earth’s atmosphere has the effect of systematically blurring images of ground-based telescopes such as the Large Synoptic Survey Telescope (LSST). In this paper, we build theoretical predictions as to how the degree of blurring varies with source wavelength. Then we obtain the same information from an atmosphere simulated in GalSim, a galaxy simulation toolkit specialized for the LSST. Comparing the results from theory and simulations serves as a necessary test for GalSim’s performance in precisely modeling the impact of atmosphere on telescope images.

Precision modeling of atmospheric effects is part of a larger goal of removing systematic bias from weak gravitational lensing inference – one of the most anticipated efforts in studying dark energy. In the following subsections, we explain the basic principles of weak gravitational lensing and its role in constraining dark energy parameters.

1.1 Principles and Applications of Weak Gravitational Lensing

Suppose a person on Earth observes light from a distant object. If there is mass in the foreground, between the observer and the object, the light arriving at Earth will have been deflected by the gravitational field of the mass, as illustrated in Fig. 1. The foreground mass is said to serve as a gravitational lens, and the object in view is said to be gravitationally lensed. Although the gravitational pull can be so strong as to create arcs and multiple replicas of the object, as in strong lensing, this is rare. Most objects in the Earth’s line of sight only undergo slight changes in their observed shapes, as in weak lensing. The changes are such that their shapes become stretched in the direction perpendicular to the direction to the lens. Fig. 2 depicts this pattern. Therefore, by studying the orientation and shapes of a large set of objects, we can infer using general relativity the gravitational field of the lens acting on them and reconstruct the distribution of foreground matter. This method is sensitive to all matter, including not only galaxy clusters but also dark matter (which does not interact with electromagnetic forces, i.e. cannot be imaged directly).

When combined with galaxy surveys, weak lensing inference has the potential to yield a three-dimensional distribution of mass in the visible universe. Note that the dimension along our line of sight represents cosmic time as well as distance, because higher-redshift galaxies are farther away and will have taken longer to arrive at the telescope than lower-redshift ones. Thus the map provides insights into the evolution of large-scale matter structures over time and serves as a cosmological probe for studying cosmic expansion driven by dark energy.
The effect of weak lensing on a single galaxy object is so small that weak lensing inference must average over a large population of galaxies with high signal-to-noise (S/N) ratios. This requirement motivates the design parameters of some upcoming surveys such as the Large Synoptic Survey Telescope (LSST), to be elaborated in the following subsection.

**Fig. 1.** Light from a distant source is bent by foreground mass[2]

**Fig. 2.** The alignment of shapes due to weak gravitational lensing. Left panels are shapes in the absence of lensing and right panels are the lensed shapes.[1]
1.2 Weak Lensing Demands on the Large Synoptic Survey Telescope (LSST)

The Large Synoptic Survey Telescope (LSST) is a telescope currently under construction in Cerro Pachón, Chile. It is projected to deliver an optical (320nm - 1050nm) survey of approximately 20,000 deg$^2$ of the night sky during its 10-year operation period. The active optics and 8.4m-diameter aperture enable high resolution, of mean blurring less than 0.7", as well as depth, of redshift up to 27.5. The LSST’s survey area and depth make the LSST dataset a prime candidate for weak lensing applications. Thanks to the big volume of usable data, statistical uncertainty on shear measurements will be low.

But a bigger data volume amplifies the effect of systematic bias. Bias becomes a problem as we attempt to extract lensing properties from images containing noise and distortions. For ground-based telescopes such as the LSST, one serious challenge is post-processing the images to correct for the atmospheric point spread function (PSF). This function governs how light from a point source is blurred as it travels through turbulence in the Earth’s atmosphere. The PSF acts on each pixel of the original object, so the observed image is the original object convolved with the PSF, as illustrated in Fig. 3. As a further example, the PSF would be a delta function centered at zero if there were to be no distortion at all.

Fig. 3. An elliptical Gaussian PSF (left), true image of a perfectly Gaussian galaxy (center), the observed image after the PSF acts on each pixel of the true image (right).

Roughly, the width of the PSF corresponds to the degree of blurring due to the atmosphere. The atmospheric PSF is anisotropic and can align the observed galaxy shapes – which can lead us to assume a lensing signal when it is not there. Even when lensing signal is present, atmospheric PSF can make it difficult to detect the signal by circularizing the lensed galaxy shapes. Lensing analysis thus requires us to build a precise model of the atmospheric PSF so that we can remove its effects from the telescope images before we infer lensing properties from them.

How precise must the model be? A study employing simulations on a catalog of galaxies has shown that the fractional variance of the atmospheric PSF must be known to approximately 0.02 in order for weak lensing analysis to be significant. Supposing that the PSF variance $r_{PSF}^2$ follows a power law relation with respect to
observed wavelength $\lambda$, as in $r_{PSF}^2 \sim \lambda^p$, (an assumption which will be motivated in Section 1.4) it is necessary to know $p$ to the precision of $\Delta p = 0.02$. [10]

1.3 The Impact of the Atmosphere on Telescope Images

How does the atmosphere distort telescope images? To see this, first suppose that astronomical sources are far enough that the wavefront originating from them is flat as it enters the atmosphere [6]. The wavefront then propagates through turbulence in the atmosphere, which consist of patches of air of varying temperatures (and thus densities) being mixed by mechanical forces, such as the wind. Note that the index of refraction in the atmosphere $n(x, t)$ can be expressed in terms of the density $\rho(x, t)$, by using the water vapor correction in the ideal gas law [16]:

$$n(x, t) \approx 1 + 0.000293 \frac{\rho(x, t)}{1.3 \times 10^{-3} \text{g cm}^{-3}},$$

where $x$ is the two-dimensional spatial vector parallel to the ground and $1.3 \times 10^{-3} \text{g cm}^{-3}$ is the density of air at 1 bar and $0^\circ$C. Note that warm air is less dense than cold air, so its refractive index is lower. Thus wave travels faster in warm air relative to cold air, resulting in a relative phase offset illustrated in Fig. 4. The turbulent mix of warm and cold air patches thus creates inhomogeneities in the refractive index that vary in space and time, and result in relative phase offsets that vary in space and time. Let a a turbulent layer of air of thickness $dh$ be given at altitude $h$ with respect to the telescope. Suppose that $dh$ is larger than the size of the eddies but sufficiently small so that diffraction within the layer will not be a problem. Then the phase fluctuations caused by the turbulent layer $\phi(x)$ is:

$$\phi(x) = \frac{2\pi}{\lambda} \int_h^{h+dh} n(x, t) dh,$$
where $\lambda$ is the wavelength of light being observed. This appends a factor $e^{i\phi}$ to the wavefunction.

In 1961, Andrei Kolmogorov developed a model of the phase aberration due to turbulence[7]. He viewed the atmosphere as a system consisting of viscous medium. Energy from solar heating or wind shear is added to the atmosphere at large scales, which he called the outer scale $L_0$. Eddies form as a result, then cascade down into smaller eddies in what is called the inertial range of the medium. Eventually, the size of the eddies reach the inner scale $l_0$, at which point they become dissipated by the friction in the viscous medium. The measured values of $L_0$ lies in the range 10m - 100m whereas $l_0$ is only a few mm. Now let the local spatial scale $l$ be given in the inertial regime, so that $l_0 < l < L_0$. For Fourier analysis, it will be more convenient to work in terms of spatial frequency $k = \frac{2\pi}{L_0}$, such that $\frac{2\pi}{L_0} < k < \frac{2\pi}{l_0}$, so we will switch back and forth as necessary. Kolmogorov derived an expression for the phase power spectrum (or the spectrum of phase fluctuations) $\Phi(k)$ which describes the phase offset at each $k$ when $L_0$ is infinity[7]:

$$\Psi(k) = B_0 r_0^{-\frac{5}{3}} |k|^{-\frac{11}{3}}$$

(3)

where the normalization constant $B_0 \approx 0.023$ ensures that the variance of phase inside the telescope aperture is 1 rad$^2$.

The assumption that $L_0$ is infinity, however, is not realistic in the case of large-aperture telescopes including the LSST (aperture diameter 8.4m). The scales of turbulence is comparable to the aperture diameter so this assumption leads us to overestimate the PSF width.[14] A more generalized version of the phase spectrum developed by von Kármán takes into account finite $L_0$:

$$\Phi(k) = B_0 r_0^{-\frac{5}{3}} |k|^2 + \left(\frac{2\pi}{L_0}\right)^2 |k|^{-\frac{11}{6}}. $$

(4)

This model, called the Kolmogorov-von Kármán model (or vK model for short), will be the model we use in our theoretical predictions and simulations.

### 1.4 The Wavelength Dependence of Atmospheric PSF

To understand the wavelength dependence of the atmospheric PSF, we must work through some derivations of the Kolmogorov model. This section is devoted to motivating the power law relation from Section 1.1:

$$r_{PSF} \propto \lambda^p,$$

(5)

where $r_{PSF}$ refers to the width of the PSF and $\lambda$ to the source wavelength.
First, we need to define what is called the phase structure function. Note that the value of the absolute phase in Equation 2 is of little interest to us; what matters is the difference of the phase at two points separated by a given distance within the inertial range, say \( l \). This motivates us to parameterize the square norm of the difference between the phases at two positions separated by \( l \), taken as the ensemble average across all of space. The structure function\[8\]

\[
D_\phi(l) = \langle |\phi(x + l) - \phi(x)|^2 \rangle_{x \in \mathbb{R}^3}
\]

is thus the spatial variance of the phase and depends only on the spatial separation \( l \), not the position \( x \). Note that, because \( \phi(x) \propto \lambda^{-1} \) in Equation 2, \( D_\phi \propto \lambda^{-2} \).

Using dimensional analysis, Kolmogorov demonstrated that the structure function follows the following power law[7]:

\[
D_\phi(l) = C_n^2(z) l^{\frac{3}{2}}, \quad l_0 < l < L_0.
\]

Here, \( C_n^2 \) is the refractive index structure constant and describes the strength of the local turbulence along the path of propagation. Fried introduced a measure of the total strength of turbulence, integrated along the path of propagation. This is called the Fried parameter, or the Fried coherence length[7]:

\[
r_0 = 0.423 \left( \frac{2\pi}{\lambda} \right)^2 \left[ \int_{\text{path}} [C_n(z')]^2 dz' \right]^{-\frac{2}{3}},
\]

where \( \lambda \) is again the source wavelength. The Fried parameter \( r_0 \) is normalized as the aperture diameter inside which the variance of phase is approximately 1 rad\(^2\). Changing variables using Equation 8 to cast Equation 7 in terms of \( r_0 \) instead of \( l \) yields\[8\]:

\[
D_\phi(r_0) = \gamma \left( \frac{|l|}{r_0} \right)^\frac{5}{3},
\]

where the normalization constant \( \gamma = \frac{2^{8/3}[R(17/4)]^2 R(21/4)}{R(11/6)R(14/3)} \approx 6.88 \) comes from the normalization for the Fried parameter. Since \( D_\phi \propto \lambda^{-2} \), it is easy to see that

\[
r_0 \propto \lambda^{6/5},
\]

since the distance \( |l| \) is independent of wavelength.

The telescope resolution is determined by the larger of the diffraction limit of the instrument with aperture diameter \( D \), \( \frac{\lambda}{D} \), and the turbulent resolution, \( \frac{\lambda}{r_0} \)[9]. This means that, for all apertures bigger than \( r_0 \), the turbulent PSF will prevent the telescope from realizing their diffraction limit; the LSST, with an aperture of 8.4m, falls under this category.
Assuming an infinite $L_0$ (Kolmogorov turbulence), we can use the approximation that $\frac{D}{\lambda} \ll 1$. Since $r_0 \propto \lambda^\frac{6}{5}$ from Equation 10, the angular resolution due to seeing is

$$\frac{\lambda}{r_0} \propto \lambda^{-\frac{2}{5}}. \quad (11)$$

Recall the wavelength power index $p$ in Equation 5 and the fact that the width of the atmospheric PSF is a measure of the turbulent resolution. Based on Equation 11, we predict $p = -0.2$ when $L_0 = \infty$. It remains to find what the equivalent of $p$ in Equation 17 is at varying $L_0$ for the von Kármán model. We do not assume that $p$ is constant; instead, let us apply the model function

$$p(\lambda) = A\lambda + B, \quad (12)$$

so that in terms of log PSF radius $\log r$ and $\log \lambda$,

$$\log r = C + (A\lambda + B)\log \lambda. \quad (13)$$

Once we obtain an expression for the atmospheric PSF, we can evaluate the width $r_{PSF}$ at each wavelength and apply the fit in Equation 13 to find $p(\lambda)$. We will continue to refer to the fit coefficients for slope and intersect – $A$ and $B$, respectively – in the proceeding sections.

## 2 Methods

The methods consisted of two main parts: (1) building theoretical predictions of $p$ from the vK model and (2) simulating the atmospheric PSF using a simulation package and obtaining $p$ from fitting Equation 13.

### 2.1 Theoretical Predictions of $p(\lambda)$

We wrote a Python program that outputs the PSF profile at a given source wavelength and outer scale. Here, we detail the steps. Recall from Section 1.2 that the observed image is the convolution of the original object by the PSF, i.e.

$$I(\alpha) = \int O(\alpha')P(\alpha - \alpha')d\alpha' = O \circ P, \quad (14)$$

where $I$ is the image, $O$ the object, $P$ the PSF and $\alpha$ refers to coordinates on the image plane. For astronomical applications, it is generally assumed that $\int P(\alpha' d\alpha' = 1$, so that the PSF only redistributes the intensity across different pixels and preserves the total intensity of the image[4].
The convolution of functions $O$ and $P$ is equal to the product of their respective Fourier transforms, which we denote by $\hat{\cdot}$, as follows:

\[ I(k) = \hat{O}(k)\hat{P}(k), \]

(15)

where $k$ refers to spatial frequencies. To simplify our calculations, we made the nontrivial assumption that the PSF was azimuthally symmetric, i.e. that it takes the same value for all polar values, so it only varies with the azimuth angle. So, from now we drop the boldfacing on the coordinate vector to collapse it into a single parameter, the azimuth angle $\alpha$, i.e. $P(\alpha) = P(\alpha)$.

The atmospheric PSF $P(\alpha)$ that we seek is the portion of the time-varying atmospheric phase that becomes captured by the telescope aperture during its exposure time. Fig. 5 illustrates the contribution of the atmosphere and the telescope aperture to the PSF. In the Fourier domain, this is a product of the atmospheric transfer function (which contains information about the random phase offsets rendered by the atmosphere, as described in Section 1.3) and the telescope’s modulation transfer function (MTF; which governs the response of the telescope aperture to the incoming wave)[8]. Let us denote the atmospheric transfer function as $A(\alpha)$ and the MTF as $\tau(\alpha)$. We have said that:

\[ \hat{P}(k) = \hat{A}(k)\hat{\tau}(k). \]

(16)

The atmospheric transfer function is defined in the Fourier domain as[8]:

\[ \hat{A}(k) = \exp \left[ -\frac{1}{2} D_\phi(\lambda k) \right]. \]

(17)

Recall that $D_\phi$, defined in Equation 6, is the spatial variance of phase.
The MTF is dependent on the parameters specific to the instrument, such as the exposure time of the telescope and the aperture shape and diameter. For simpler calculations, we nontrivially assumed that the exposure time was infinite and that the aperture was circular with diameter \( D = 8.6\text{m} \). In this case, the MTF is defined in the Fourier domain as [8]:

\[
\hat{\tau}(k) = \left(\frac{2}{\pi}\right) \left[ \arccos \left( \frac{k}{D} \right) - \frac{k}{D} \sqrt{1 - \left( \frac{k}{D} \right)^2} \right].
\] (18)

The PSF is then

\[
\hat{P}(k) = \exp \left[ -\frac{1}{2} D_\phi(\lambda k) \right] \left(\frac{2}{\pi}\right) \left[ \arccos \left( \frac{k}{D} \right) - \frac{k}{D} \sqrt{1 - \left( \frac{k}{D} \right)^2} \right].
\] (19)

We numerically computed the inverse Fourier transform of Equation 19, which yielded an expression for the PSF in spatial coordinates, \( P(\alpha) \). Note that \( P(\alpha) \) also depends on wavelength \( \lambda \) which show up in both \( A \) and \( \tau \) and the outer scale \( L_0 \) which show up in \( A \). Evaluating \( P \) along the full range of \( \alpha \), with \( \lambda \) and \( L_0 \) fixed, gives the spatial PSF profile.

In order to find the wavelength power index \( p(\lambda) \), we took the full-width half maximum (FWHM) of the PSF profile as a measure of the PSF width \( r_{PSF} \) in radians. Then we evaluated \( r_{PSF} \) at wavelengths from 300nm to 1100nm and performed a fit using Equation 13 to obtain \( p(\lambda) \). This process was repeated for \( L_0 \) values of 25m, 50m, 75m, 100m, 1000m and infinity.

2.2 Simulations in GalSim

We simulated the atmosphere PSF using GalSim, a modular galaxy image simulation toolkit specialized for simulating the LSST dataset. [4] A module representing the atmospheric PSF has recently been added to the GalSim package. Using the atmosphere module involves a series of steps that model the passage of light through the atmosphere onto the telescope’s focal plane. First, an atmosphere is created from two-dimensional phase screens. Because mapping a three-dimensional time-evolving atmosphere is computationally expensive, GalSim operates by creating multiple two-dimensional phase screens \( \Psi_i(x, y) \) at different altitudes and taking the weighted average of the screens. Fig. 6 illustrates six layers of phase screens used in GalSim. Each phase screen in the spatial domain, \( \Psi_i(x, y) \), is computed by taking the inverse fast Fourier transform (FFT) of the von Kármán phase spectrum in the frequency domain, \( \Psi_i(k) = \Psi_i(u, v) \) which was given in Equation 4. To wit:

\[
\Psi_i(x, y) = FFT^{-1}[\Psi_i(u, v)].
\] (20)

In creating each screen, GalSim accepts atmospheric parameters such as the outer scale \( L_0 \), wind speed/direction, size of the screen array in physical units and the resolution of the screen array. In addition, the \( r_0 \) at 500nm is accepted so that GalSim can internally convert it at other source wavelengths using the relationship \( r_0 \propto \lambda^{6/5} \).
given in Equation 10. The wind speed and direction may be generated pseudo-randomly, so that they are different for each screen. They represent the speed at which phase screens evolve in time. Once each phase screen is created, Galsim takes the weighted average $Ψ$ of the screens $Ψ_i$ at each position $(x,y)$ of the image plane, as follows:

$$Ψ(x,y) = \sum_i c_i Ψ_i(x,y), \quad \sum_i c_i = 1. \quad (21)$$

To approximate seeing conditions at the LSST site, we used the measurements taken in Cerro Pachón, Chile, where the Gemini-South telescope is located and the LSST is being built.[19] The measurements yield six phase screens of altitudes 0, 2.58, 5.16, 7.73, 12.89, and 15.46 km above the telescope altitude, with relative weights of 0.652, 0.172, 0.055, 0.025, 0.074, and 0.022, respectively.[5] The measured median value of 0.16 m is used for the Fried parameter $r_0$.[19] The maximum wind velocity at all the phase screens altitude is set as 20 m/s. Ziad (2000) gives the measured outer scale as 20m, although the outer scale will be varied throughout our analysis to confirm the agreement between theory and simulations.[20] This means that, in order to meet the Nyquist sampling rate, the time step should be at least 0.005 s.[5] Note that the time evolution of the phase screens did not matter when building the theoretical predictions in Section 2.1, because we assumed infinite exposure. In contrast, for GalSim where the computation time scales with exposure time, we must input a finite exposure time. The wind speeds and directions represent the speeds at which the phase screens evolve in time so it is important to ensure that sizable set of instantaneous PSFs can be sampled over the exposure time given the wind speeds and directions.

The second step is to define the telescope aperture, by specifying the pupil-related parameters such as the aperture diameter and fraction of central obscuration, which is nonzero for an annulus-shaped aperture. Fig. 7 gives a schematic picture of the LSST’s annulus-shaped aperture; it has a fractional central obscuration of 0.6. Although we used the LSST pupil parameter of diameter 8.4m, we set the central obscuration to zero in order to approximate the conditions in the theoretical predictions of Section 2.1. Given these pupil parameters, GalSim
internally computes the MTF $\tau$, given in Equation 18.

Once the aperture is created, we may finally project the aperture through the phase screen and construct the PSF. We also specify the wavelength $\lambda$ of observation, the period of time over which to integrate the PSF (the exposure time) as well as the time step during which the screens are allowed to evolve. As for the total exposure time, note that, more instantaneous PSFs are sampled with longer exposure time so we see less irregularities in the drawn PSF. Fig. 8 illustrates this phenomenon. Increasing the exposure time reduces uncertainty in width estimation so we choose a relatively long exposure time of 30s; this is also the LSST value.

The final step is drawing the PSF, making sure to provide a canvas big enough so that all of the PSF intensity (summing to 1 over the entire image array) is captured and resolved enough so that image analysis algorithms can be called on the image, to yield measurements with sufficient precision. Note that azimuthal symmetry is not assumed when using GalSim. Thus it is possible for the PSF to take the shape of an ellipse, or even a non-parametric shape. Algorithms for finding FWHM for non-parametric shapes can be inadequate because FWHM will differ dramatically depending on which half-maximum pixels are sampled. So we use the second central moment of
the entire image as a measure of the PSF width \( r_{PSF} \). The moment estimation algorithm is included in the GalSim package, and operates by using adaptive moments to fit an elliptical Gaussian to the image. For the telescope field of view to be within the moving phase screens during the 30 s of exposure time, the phase screen size must be approximately \( 1.3 \text{ km} \times 1.3 \text{ km} \)[5]. The resolution of the phase screen must be some fraction of \( r_0 \) – we choose 0.04 m per pixel. This determines the size of the phase screens as 32,500 x 32,500 pixels.

Because the GalSim simulations are based on randomly evolving phase screens, we need an estimate of the error, i.e. the variance of the PSF profile across multiple atmospheres. So we repeat the simulation for 50 atmospheres created with different random seeds. Although a larger sample would give us a better handle on the error, a sample size of 50 was chosen for computational speed considerations. To obtain \( p(\lambda) \) from the GalSim simulation results, we apply the fit in Equation 13 for each of the 50 runs, then take the mean of \( A \) and \( B \) as our final fit coefficients. The fitting errors are added in quadrature across the 50 runs. This process was repeated for \( L_0 \) values of 25m, 50m, 75m, 100m, 1000m and infinity.

3 Results

In this section, we report the theoretical predictions of the wavelength power index \( p(\lambda) \) and the \( p(\lambda) \) from the simulations in GalSim.

3.1 Theoretical Predictions

Fig. 9 gives the log-log plot of the theoretical PSF width \( r_{PSF} \) in radians vs. the source wavelength \( \lambda \) in nm across varying outer scales, overlaid with the fit described by Equation 13. The legend gives the fitted \( p(\lambda) \) for each outer scale.

Fig. 10 plots the theoretical predictions of the wavelength power index \( p(\lambda) \). The fitting error was numerical, on the order of 1e-27, so we neglect the errors on the fit coefficients of the theoretical predictions. Note, in particular, that \( p = -0.2 \) for infinite outer scale which was the prediction we made in Section 1.4.

3.2 Simulations in GalSim

Fig. 11 gives the log-log plot of the simulated PSF width \( r_{PSF} \) in pixels vs. the source wavelength \( \lambda \) in nm across varying outer scales, overlaid with the fit described by Equation 13. As described in Section 2.2, each
Fig. 9. Theoretical PSF width vs. wavelength for varying outer scales. Legend gives the fitted $p(\lambda)$.

Fig. 10. Theoretical power index vs. wavelength for varying outer scales

Point in Fig. 11 carries error approximated by the sample standard deviation across the 50 runs. It was omitted from Fig. 11, however, for an uncluttered view of the plot. Note that the PSF width outputted by GalSim is in units of pixels, so the scaling of the y-axis in Fig. 11 differs from that in Fig. 9.

Fig. 12 plots the simulated wavelength power index $p(\lambda)$. In contrast to Fig. 10, the fitting errors were not negligible so the bands in Fig. 12 are the error bounds on $p(\lambda)$ propagated from the errors on the fit coefficients $A$ and $B$ in Equation 12.
4 Analysis: Comparison of Theory and Simulations

We compare the wavelength power index $p(\lambda)$ between theory and simulations. Figures 13 and 14 overlay the plots in Figures 10 and 12.

For outer scales 75m, 100m, 1000m and infinity, $p(\lambda)$ values agree. But at the two lowest outer scales 25m and 50m, we find disagreement. We also see that the error bands were 25m and 50m are the widest of all outer scales, i.e. the fitting errors were highest.
Recall that the PSF width for the PSF generated in GalSim were approximated by the second moment of the PSF image, returned by GalSim’s internal image analysis algorithm. Given that the PSF widths at 25m and 50m are the smallest of all outer scales, the disagreement between theory and simulations at 25m and 50m may simply be the failure of the algorithm at small PSFs. Because the algorithm averages over fewer pixels for small images, the reported second moments may be less accurate. The algorithm does not report the error bounds on the measured moments, so it is difficult to check whether it was sufficiently accurate. In order to explore the accuracy of the algorithm, we recommend using a higher image resolution at lower outer scales so that the algorithm can average over more pixels.
Alternatively, the disagreement may be due to real-world parameters that were not matched between theory and simulations. For instance, while the theory assumed infinite exposure, the 30s of exposure time in GalSim may not have been long enough to sample over sufficiently many instantaneous PSFs.

5 Preliminary Results for Geometric Monte Carlo Simulations

GalSim currently offers the option to use geometric approximation, or ray tracing, when projecting the atmospheric phase screen onto the telescope aperture. This means that the PSF is generated by tracing the path of each photon from the atmosphere through the pixels on the image plane, i.e. the angles of incident photons are calculated from the phase screen gradient. Then they are drawn into an image using a Monte Carlo sampling method (called photon shooting). Under the photon shooting method, the PSF is registered as a two-dimensional probability density function from which a finite number of photons are generated. Each photon occupies one whole pixel. Photon shooting is about 20 times faster than direct (analytical) rendering for creating images with dimensions 128 pix $\times$ 128 pix with $10^6$ photons. Speed is crucial for simulating atmospheric PSFs, especially as telescopes of LSST’s size typically require a large canvas, of at least 8192 pix $\times$ 8192 pix with pixel scale smaller than 1 m per pixel.[5]

We have said that, in the image rendering step, photon shooting is much faster than direct rendering. Calculating the PSF via geometric approximation also speeds up computation by a magnitude of 5 relative to the analytic FFT convolution. The larger and the more resolved the image array, more exposure time may be required – in that case, geometric approximation followed by photon shooting is the method of choice because its computation time does not scale with exposure time, only with the number of photons being shot. In fact, it is preferred over the analytical convolution followed by direct rendering for scientific reasons as well, because it allows us to build a more precise model of the real scenario – of a photon landing on the telescope’s focal plane. It can accommodate analyses of effects due to photon interactions coming from the charge-coupled sensors on the focal plane. Given such benefits, it is worth testing the performance of geometric photon shooting simulations to check if further debugging is necessary.

5.1 Implementation of Wavelength-Dependent Monte Carlo Image Rendering

Because the atmospheric PSF varies with wavelength, in order to implement the geometric approximation, it is necessary to be able to draw objects at varying wavelengths via photon shooting. Previously in GalSim, photon shooting was only available for monochromatic objects.
In order to implement photon shooting for drawing wavelength-dependent objects (called **chromatic photon shooting**), I first multiplied the spectral energy distribution (SED) of the chromatic object by the LSST bandpass throughput, to obtain the flux reaching the telescope at each wavelength. Normalizing this figure to unity yielded the probability density function from which wavelengths of photons could be randomly sampled, \( p(\lambda) \). On the other hand, normalizing the figure to number of photons (an input parameter \( n \) representing the total flux) yielded the number of photons to shoot at each wavelength, \( F(\lambda) \). Then we ran the monochromatic photon shooting \( F(\lambda) \) times at \( \lambda \) values randomly generated from \( p(\lambda) \). This allowed me to implement chromatic photon shooting in GalSim. My work with chromatic photon shooting was incorporated into the geometric approximation module of GalSim.

### 5.2 The Dependence of PSF Width on Wavelength and Outer Scale

Fig. 15 gives the plot of geometrical PSF width vs. wavelength for various outer scales. For all outer scales, the radius appears constant across wavelengths. To confirm this observation, we apply a log-log fitting as we did for the analytical simulation, the results of which are given in Fig. 16. Indeed, there seems to be minimal dependence on wavelength. We plot the \( p(\lambda) \) across wavelength in Fig. 17-18.

At all outer scales, the results are consistent with \( p(\lambda) = 0 \).

![Fig. 15. Plot of PSF width in pixels vs. wavelength in nm](image)
Now we proceed to comparing the simulations generated via analytic convolution and via geometric photon shooting. For speed considerations, only 20 runs were repeated for each data point. The null hypothesis should be that the PSF widths are the same, as each of the 20 runs of the analytical and geometrical simulations are generated from the same atmosphere. Figures 19-20 overlay the two PSF widths across wavelengths.

Interestingly, at all wavelengths and at all outer scales, the geometrical PSF width is larger than the analytical PSF width. Finally, we include images of the geometrical and analytical PSFs for a visual check; indeed the geometrical PSF appears more smeared.
Fig. 18. Plot of the wavelength power index vs. wavelength for geometrical photon shooting at outer scales 25m, 50m, 75m

Fig. 19. Plot of geometrical and analytical PSF widths in pixels vs. wavelength in nm, at outer scales 100m, 1000m, infinity

Fig. 20. Plot of geometrical and analytical PSF widths in pixels vs. wavelength in nm, at outer scales 25m, 50m, 75m
6 Conclusion and Suggestions for Further Research

We have built theoretical predictions of the atmospheric PSF profile and simulated the atmospheric PSF profile using the analytic module of the simulation package GalSim, and compared the dependence of the PSF width on source wavelength. At the two lowest outer scales we have explored, 25m and 50m, the theory and simulations do not agree. We have attributed this to incompleteness of the model and unknown precision/accuracy of the width estimation algorithm. At outer scales 75m, 100m, 1000m and infinity, they do agree. On the other hand, the geometric approximation module of GalSim appears to be incapable of reproducing the theoretical wavelength-dependence of PSF width, reporting a PSF width that is constant across wavelength and greater than the width of the analytically-simulated PSF at all wavelengths.

Although we have some confidence in the performance of the analytic module of GalSim for simulating wavelength-dependent atmospheric effects, we must further evaluate it for smaller outer scales – especially as 25m is closest to the measured value at the LSST site, of 20m. The comparisons made in this paper serve as a good test of GalSim for the larger goal of removing bias from the atmosphere in the context of using the LSST dataset for weak gravitational lensing.

In order to assess whether the power law discrepancy between theory and simulations is an artifact of the second-moment estimation algorithm or the result of not matching enough parameters between theory and simulations, we would have to extend the theoretical model to include dependence on real-world parameters, such as the telescope exposure time.

Moreover, a bulk of the analysis in this paper depends on the performance of GalSim’s second-moment estimation algorithm. The algorithm is called on the image of the PSF, so it remains to find what the optimal image
size and resolution is for a given set of atmosphere and aperture parameters. It would also benefit our analysis to obtain an upper bound on the error of radius estimation. If, for instance, the error on the reported PSF width is comparable to the differences of PSF width across wavelengths and across outer scales, the fitting we applied to obtain the power index $p(\lambda)$ would not be meaningful. Even if the error is small, it may depend on the size of the PSF – which contributes systematic bias to our analysis on wavelength-dependence, as PSF width at longer wavelengths are smaller.

Lastly, it would be interesting to explore whether other parameter of the analytical and geometrical PSFs, such as the ellipticity, agree at any outer scale or wavelength. It will serve as a key with which to determine whether the wavelength-independence of geometrical PSFs is the result of a bug or speaks to a fundamental limitation of the ray tracing algorithm.
References

15. Lucke, Robert L., and Cynthia Y. Young. "Theoretical wave structure function when the effect of the outer scale is significant." Applied optics 46.4 (2007): 559-569.