Angular Distribution of Events in Water Cherenkov Detectors for Supernova Neutrino Measurements

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Abstract

This study explores the angular distribution of supernova neutrino events in water Cherenkov detectors. The results of such a study will be important in determining the flavor content of the neutrinos emitted from a supernova burst, and it will improve our understanding of detector response to the direction of incoming neutrinos. In particular, neutrino-electron elastic scattering may help point to supernovae. Monte Carlo simulations of supernova burst neutrinos will be used in order to study the angular distribution expected, which will help us understand how the detector will respond when an actual supernova occurs.
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2 Introduction

2.1 Supernovae

Supernovae are among the most brilliant and intense astronomical events in the cosmos. Early astronomers could see some supernovae with the naked eye and were fascinated by them as far back as the year 185 CE [2]. Today, physicists study supernovae to understand how these explosions originate and how they affect their surrounding interstellar medium. So far, modern astronomers have been able to study supernovae through observations of events occurring in other galaxies. Although astrophysicists have made great progress in understanding supernovae and the process of how they originate, there are still significant gaps in our understanding of the explosion mechanism, especially concerning core collapse.

Supernovae may be categorized by the spectrum of matter emitted from the explosion. Observations of supernovae in other galaxies have revealed that there are two main types known as Type I and Type II, although supernovae may share some characteristics of both. Type I supernovae consist of matter spectra
that lack identifiable hydrogen, while Type II supernovae spectra contain a normal abundance of elements [3]. In addition, within each supernova type, differences in the light output curves indicate distinctive subcategories. The category of a supernova results from the nature of the star that generated it. Stars in a binary system may evolve into Type Ia supernovae under certain conditions. Type II and Ib/c supernovae may be produced by the core collapse of enormous stars with a mass range that is roughly between nine and fifty solar masses. These types are relevant to this study because a large neutrino burst is emitted during the core collapse process. Stars with very large masses presumably turn into black holes. Understanding how mass is related to the core collapse process is an essential part of explaining how a supernova occurs.

Although there is still much to figure out about core collapse, it is clear that the process mainly arises through the conflict between gravity and other forces that prevent a star from collapsing. The gravitational force of a massive star draws matter toward the center and potential energy is converted into other forms. Initially, nuclear fusion produces enough thermal energy to support the mass of the star from collapsing inward. The fusion reactions result in heavier nuclei and over time produce layers of progressively heavier elements, as depicted in Figure 2. Eventually, the creation of heavier elements produces an iron core, which no longer results in the release of binding energy, and matter is only prevented from collapsing by electron degeneracy pressure. When the gravitational force exceeds the degeneracy pressure, there is a sudden collapse in which the protons in iron nuclei are converted into neutrons through inverse beta decay. An enormous amount of energy is released through a burst of neutrinos that lasts merely seconds. Soon the collapse ends as matter encounters neutron degeneracy pressure and a repulsive strong force interaction due to the dense arrangement of matter. Once this occurs, the collapsing matter is pushed outward as a shock wave and the outer layers of nuclei are sent outwards and form the supernova explosion.

The low energy neutrinos of a supernova are emitted for about ten seconds and there is an equipartition of energy among all flavors. The flux is higher during the first few seconds, and the neutrinos release a total energy on the order of $10^{53}$ ergs with energy ranging from a few to tens of MeV. A rough calculation suggests that a few hundred of these neutrinos per kiloton of detector may be observed on Earth [4]. Due to the nature of the weak interaction, the neutrinos diffuse much more rapidly than the surrounding infalling matter. Photons also may be observed as they are emitted from supernovae, and in the future, even gravitational waves may be detected from these sources; however, supernova neutrinos are able to escape within tens of seconds while photons might take hours. Thus, supernova neutrinos can provide a valuable early warning of an event. This is especially important because supernova events are so rare that observing a supernova as it occurs is difficult. Galactic supernovae are likely to occur only a few times per century. Most often, supernovae are only discovered when they are already in progress, which may result in wasting a huge opportunity for supernova astronomy.
Figure 2: The various layers that appear in an evolved star before a supernova occurs [7].

The concept of detecting neutrinos from a supernova event was put to experimental test when SN1987A was observed. SN1987A occurred approximately 51 kpc away from Earth in the Large Magellanic Cloud and is significant in that it is the only supernova for which neutrino events were detected [5]. A total of 24 neutrino events were observed using the Kamiokande-II, IMB, and Baksan detectors [6]. Although an extremely limited number of events were detected, the energy and time distributions were consistent with theoretical models. The number of neutrino events observed is encouraging, because it indicates that a supernova occurring within our galaxy would produce enough data to locate it using neutrinos.

2.2 Neutrino Detection

Detectors are able to provide information about supernovae by indirectly observing the time, energy and flavor of incoming supernova neutrinos [4]. These properties may only be discerned with difficulty because neutrinos are electrically neutral, and it is challenging to measure the incredibly small forces of the weak interaction and gravity [3]. Thus, these ghostlike neutrinos typically pass through nearly all material encountered without a single interaction occurring. Neutrino detectors rely on observing the effects of particles that are emitted when a neutrino does interact.

Water Cherenkov detectors take advantage of the fact that neutrino interactions occasionally produce
charged particles, which are much more easily detectable. These detectors are often designed to contain a huge volume of water to account for the low cross section of neutrino interactions. Using a larger volume of water provides more targets for incoming neutrinos, which naturally increases the number of detectable interactions. The interactions of interest all involve the production of charged particles as output. If the initial neutrino arrives with enough energy upon interaction, the resulting charged particle may travel at a velocity that exceeds the speed of light in water. When this occurs, photons are emitted at an angle to the direction of the charged particle shown in Figure 3. This phenomenon, known as Cherenkov radiation, results in a cone-shaped pattern of light that is analogous to the shape created by the propagation of sound waves during a sonic boom.

The characteristic Cherenkov radiation produced after a neutrino interaction may be measured to reconstruct information about the neutrino. The inner surface of a Cherenkov detector is covered with photomultiplier tubes in order to measure photons travelling outward in all directions. These photomultiplier tubes directly observe photons by measuring charge caused upon impact and recording the time at the moment of observation. The charge collected by the photomultiplier tube is proportional to the number of photons, which is proportional to the energy lost by the particle. The geometry of the photomultiplier tubes within the tank is known, so the shape and direction of the Cherenkov ring and consequently the direction and energy of the particle may be inferred from the distribution of photons observed. Fitters are algorithms that use the photomultiplier tube observations to reconstruct information about the particle. The BONSAI fitter computes the lepton position and momentum given the photomultiplier tube hit, charge and time data, and it is optimized for low energy events.

The Super-Kamiokande water Cherenkov neutrino detector is examined in this study. The detector, located deep below Mount Ibanoi, consists of a cylindrical tank containing approximately fifty kilotons of pure water surrounded by photomultiplier tubes [9]. The detector is one kilometer underground, which reduces the intensity of the cosmic muon background by a factor of approximately $10^{-6}$. Ultra-pure filtered
groundwater is used in order to reduce background noise from radon decay and scattering. The detector background interaction rate of Super-Kamiokande is low enough to easily satisfy the criteria of detecting the signal of a supernova \cite{4}. In addition, the ability of the detector to provide information about incoming neutrinos may allow the direction of the supernova to be discerned.

Thus, water Cherenkov detectors may provide some information about neutrinos by observing the properties of leptons emitted within the tank; however, the actual neutrino information determined depends on the type of interaction that occurs. A neutrino may interact through several channels in a water Cherenkov detector. For low energy neutrinos in the few-\text{tens-}of\text{\,MeV} range, the largest cross section comes from inverse beta decay because of the abundance of free protons \cite{11}. Inverse beta decay occurs when a neutrino interacts with a proton and results in the release of a lepton and a neutron, as shown in the following equation:

\[ \nu_e p \rightarrow e^+ n \]  \hspace{1cm} (1)

A useful approximation for the cross section has been accurately computed and shown to be valid at relevant supernova energies \cite{12}:

\[ \frac{d\sigma}{d\cos \theta}(E_\nu, \cos \theta) \approx \frac{G_F^2 m_p p_e \varepsilon \cos^2 \theta_c}{\pi (s - m_p^2)^2 \left(1 + \varepsilon (1 - \frac{E_\nu}{p_e} \cos \theta)\right)} |\mathcal{M}|^2 \]  \hspace{1cm} (2)

- $G_F^2$ is the Fermi coupling
- $\cos \theta_c$ is the cosine of the Cabibbo angle
\( \varepsilon = E_\nu/m_p \)

\( E_e = \frac{(E_\nu - \delta)(1 + \varepsilon) + \varepsilon \cos \theta \sqrt{(E_\nu - \delta)^2 - m_e^2}}{\kappa} \), with \( \kappa = (1 + \varepsilon)^2 - (\varepsilon \cos \theta)^2 \)

\( p_c = \sqrt{E_e^2 - m_e^2} \)

\( M \) is the current-current structure, where:

\[
|M^2| = A(t) - (s - u)B(t) - (s - u)^2C(t)
\]

\( A \approx M^2(f_1^2 - g_1^2)(t - m_e^2) - M^2\Delta^2(f_1^2 + g_1^2) \)

\( B \approx 0 \)

\( C \approx (f_1^2 + g_1^2)/4 \)

\( \{f_1, f_2\} = \frac{(1 - (1 + \zeta)(t/M^2_\lambda)^2)}{(1 - t/M^2_\lambda)}, \quad g_1 = \frac{g_1(0)}{(1 - t/M^2_\lambda)^2}, \quad g_2 = \frac{2M^2g_1}{m_e^2 - t} \)

\( \Delta = m_n - m_p \)

\( M = (m_p + m_n)/2 \)

\( M^2_\lambda = 0.71 \text{ GeV}^2 \)

\( M^2_\lambda \approx 1 \text{ GeV}^2 \)

\( \zeta = \kappa_p - \kappa_n = 3.706 \)

\( s - m_p^2 = 2m_pE_\nu, s - u = 2m_p(E_\nu + E_e) - m_e^2, t = m_n^2 - m_p^2 - 2m_p(E_\nu - E_e) \)

\( s = 2m_pE_\nu + m_p^2 \)

\( u = s - 2m_p(E_\nu + E_e) + m_e^2 \)

\( t = m_n^2 - m_p^2 - 2m_p(E_\nu - E_e) \)

The equation for the cross section allows distributions of useful kinematical qualities to be computed. In addition, the lepton energy may be approximated by the following equation:

\[
E_e \approx E_\nu - 1.8 \text{ MeV}
\]

The equations describing inverse beta decay show that the distribution of outgoing leptons is nearly isotropic at lower energies, and thus are not the most effective method to point to a supernova. Instead, the best reaction to examine is neutrino-electron elastic scattering:
\[ \nu_x + e^- \rightarrow \nu_x + e^- \] \hspace{1cm} (5)

The neutrino-electron elastic scattering cross section is described by the following equation:

\[ \frac{d\sigma}{dE_e}(E_\nu, E_e) = g_l^2 + g_r^2(1 - \frac{E_e}{E_\nu})^2 - g_l g_r m_e E_e E_r^2 \] \hspace{1cm} (6)

- \( g_l = \theta_w + 1/2 \)
- \( g_r = \theta_w \)
- \( \theta_w = 0.231 \) (the Weinberg angle)

The final electron energy and scattering angle are related by the following equation obtained through simple relativistic kinematics:

\[ \cos \theta = \frac{E_e(E_\nu + m_e)}{E_\nu \sqrt{E_e + 2m_e}} \] \hspace{1cm} (7)

Details of the calculation are given in Appendix A. The value of theta refers to the angle between the paths of the incoming neutrino and the outgoing electron.

The directionality of the neutrino-electron scattering interaction is especially crucial to the study of supernovae. The distribution of electrons resulting from the scattering is peaked in the forward direction of the incoming neutrino. Thus, depending on the angular resolution of the detector, directional data may be able to point out supernovae.

The angular resolution of the detector is an important quality to know when fitting data because it is one of the factors that limits the ability to discern directionality. Angular resolution is a measure of how closely the fitted direction compares to the actual direction at a particular energy. It is defined as the angle from the true particle direction that 68% of fitted events fall within. This quantity may be probed using the Monte Carlo method, which in this case involves simulating many particles in the detector, computing the angle between each original particle’s true direction and the fitted direction, and using the resulting distribution to find the angular resolution.

In this study, the Monte Carlo method is used to probe the angular response of a water Cherenkov detector with the configuration of Super-Kamiokande to inverse beta decay and neutrino-electron scattering interactions. The directional resolution of leptons will first be studied for simulations created using simulation software SKDetSim. The directional resolution will be determined by fitting simulations of electrons with BONSAI fitter. After studying the ability of the fitter to discern the direction of leptons, a similar procedure
will be applied to study the directional resolution of neutrinos interacting within the tank. Neutrino-electron elastic scattering and inverse beta decay interactions in particular will be studied. Angular resolution curves will be created for both of these interactions. Finally, smearing plots and matrices will be created containing information about the normalized angular distributions of fitted directions at energies relevant to supernova detection.
3 Results

3.1 Electron Angular Resolution Study

Monte Carlo simulations may be used in order to study the angular resolution of the detector. In this study, the Monte Carlo method involves simulating how the detector responds to particles and then comparing to the original information to check how well the properties of the particle may be inferred using only information gained from the detector. In this case, electrons are simulated within the tank and the fitted position and direction are compared to the known position and direction of the simulated Monte Carlo electrons.

The simulations are created using software designed for simulating the physics of Water Cherenkov detectors. In this part of the study, simulations created using two simulation packages, SKDetSim and WCSim. These programs accept kinematic information of particles as input and create an output file containing information of each photomultiplier tube’s charge and time response for every event. The kinematic information consists of a list of particles to simulate containing the type, direction, position and energy of each particle in the event. SKDetSim is the well-understood detector simulation package used for Super-Kamiokande, while WCSim is a newer, Geant4 based program that may be run with the photomultiplier tube configuration of Super-Kamiokande. Figure 5 shows an event display of a 20 MeV electron simulated using SKDetSim and viewed using Superscan, an event display program. The event display represents the cylindrical configuration of the detector projected onto a two dimensional image, with each colored dot representing a photomultiplier tube that recorded a certain charge. The smaller display in the upper right corresponds to the inner detector, which did not measure any hits in this example.

In this study, monochromatic electrons were studied using the detector configuration of Super-Kamiokande. For a particular energy, three hundred electrons were simulated uniformly within the tank and given a random direction. The detector output was then studied using the BONSAI fitter. BONSAI (Branch Optimization Navigating Successive Annealing Iterations) is a subroutine that fits the vertex and direction of low energy events using relative PMT hit timing as input [10]. The original vertex and direction given to the simulated particle are known, so for each electron the fit may be compared. Figure 6 shows the distributions of the distance from the fitted vertex to the true vertex for 20 MeV electrons with SKDetSim on the left and WCSim on the right.

The dot product between the fitted and true direction vectors was also computed for 20 MeV electrons in Figure 7.

The distributions show that both the fitted vertex and the fitted direction match more closely to the true values of vertex and direction for SKDetSim simulations. For a better vertex fit, the difference between the
Figure 5: Event display of a 20 MeV electron.

Figure 6: Distributions of distance (cm) between true electron vertex and BONSAI fitted vertex using SKDetSim (left) and WCSim (right).
fitted and true vertices should closer to zero, because zero difference in distance would indicate an exact fit. For SKDetSim, BONSAI is most likely to give a fit that is around 20 cm off from the exact truth position. The wider curve and higher mean of the WCSim distribution indicate a poorer fit. For a better fit, the dot product distribution should be closer to one, which corresponds to an exact fit for the direction. Noting the scale of the plots, it is clear that the mean of the SKDetSim distribution is much closer to one, and the WCSim distribution is wider. The angular resolution may be obtained using the Monte Carlo angular difference distribution. In this study, the angular resolution is defined as the angle within which 68% of events are contained. This may easily be computed by integrating the distribution from zero to the angle at which the number of events integrated comprises 68% of the total number of events. Note that the above plots show the distribution of dot products, rather than the distribution of angular differences. The angular resolution is computed for a selection of energies and shown in Figure 8.

Both plots show the same general trend of decreasing angular resolution value as the electron energy is increased. This occurs because the energy is proportional to the number of photons emitted, and an increase in photons means there are more statistics with which the direction and vertex of the electron may be fitted. However, noting the scale of the plots, it is clear that the angles are significantly greater for the WCSim Monte Carlo. A previous study on solar neutrinos in Super-Kamiokande produced the curve shown in Figure 9 describing electron angular resolution in the detector.

The angular resolution curve made using SKDetSim corresponds to the previous study, while the angular resolution predicted by WCSim is poorer. Differences between the two programs are expected; SKDetSim has been studied for several years and simulations closely correspond to physical behavior observed within
Figure 8: Plots of angular resolution for electron simulations made using SKDetSim (left) and WCSim (right).

Figure 9: Angular resolution plot from previous solar neutrino study [13].
Figure 10: Generating the kinematics of the lepton direction vector from the initial neutrino direction.

the detector; however, work is still in progress on the newer WCSim program. Thus, SKDetSim will be used throughout the rest of this study.

3.2 Elastic Neutrino-Electron Collision Study

A similar method will be used to study the angular resolution of elastic electron collisions. The main difference from the previous investigation is that here, the kinematics of neutrino-electron elastic scattering interactions are used to generate the electrons, and the reconstructed information is compared to the true direction of the original neutrino. The distribution of angular differences will then reveal how well the interaction may be used to point out the direction of detected neutrino events.

The original distribution of neutrinos is first generated in the same manner as the electrons from before; at each energy considered, three hundred neutrinos are simulated uniformly within the tank and given isotropic direction vectors. The electron cross section as a function of energy is then used as a probability distribution to randomly select the energy of the emitted electron. The kinematics equation determines the angle between the original neutrino direction vector and the resulting electron direction vector, which is shown in Figure 10. These values allow the kinematic information of the electron to be determined: an arbitrary azimuthal angle is chosen, and the resulting electron direction vector is obtained by rotating the neutrino direction vector by the angle computed from the kinematics. As a result, information about the original neutrino directions is saved and a kinematics file is produced for the resulting electrons. The electrons are then simulated with SKDetSim and fitted using BONSAI as before, and the fitted directions are compared to the original neutrino directions. Figure 11 shows the distribution of dot products between the fitted electron directions and the true neutrino directions.

The dot product distribution is much less sharply peaked than the corresponding distribution comparing
fitted electron directions to the true electron directions. The values of dot product obtained for the simulated neutrino-electron elastic scattering are generally more spread out than the dot products for true electron direction because the collision sends the electrons off at angles. Figure 12 shows the angular resolution obtained for several energies.

The plot shows the same general trend as earlier plots of electron angular resolution. As the initial neutrino energy is increased, the angular resolution improves. This is expected because the differential cross section is more strongly peaked in the initial neutrino direction as energy is increased. It is also apparent that the change in resolution is lower when considering higher energies, which indicates that the ability of the reaction to point forward improves as energy is increased.

It is useful to summarize the overall angular response of the detector to a reaction across all relevant supernova energies in the form of a “smearing matrix.” In this matrix, each column corresponds to a particular energy and contains all the entries of its normalized angular difference distribution. Thus, the entire matrix characterizes the angular response to the reaction of interest and shows how the angular difference distributions vary across energies.

The information contained within the smearing matrix may also be expressed as a three dimensional plot. For elastic electron collision reactions, energies at intervals of 0.5 MeV from 0.5 MeV to 100 MeV were considered. At each energy, the angular difference distribution (expressed as cosines of the angles) was obtained using Monte Carlo as before. Each distribution was then normalized such that the integral across all angles equals one, and then fitted with the following function to obtain a smooth curve describing the distribution:
Figure 12: Angular resolution of true neutrino direction for elastic electron collision simulations created using SKDetSim.

\[ f(x) = a_0e^{a_1x} + a_2 + a_3x + a_4x^2 \]  

(8)

The Monte Carlo sample is not large enough to obtain a smooth distribution, so it is useful to examine the results with these fitted functions. Examples of normalized distributions for 20 MeV, 60 MeV, and 80 MeV neutrinos are shown with the fitted function in Figure 13. The plots show how the directionality increases as the initial neutrino energy is increased. This fitting process is done for the entire range of energies considered, and the functions are evaluated at the appropriate values to produce Figure 14. To create the corresponding smearing matrix, the fitted functions of the normalized distributions are evaluated and entered into a text file. This smearing matrix is expressed in a form that may be used in the SNOwGLoBES software package to simulate supernova neutrino burst distributions. For example, if the matrix multiplies a vector containing the distribution of neutrino energies, the result is the overall angular distribution that would be observed in the detector.

3.3 Inverse Beta Decay Study

The other reaction considered in this study is inverse beta decay. The procedure to examine this reaction is nearly identical to the method followed for elastic electron collisions: Monte Carlo simulations of leptons resulting from inverse beta decay interactions are used to probe the angular resolution. The inverse beta decay reaction has a different cross section, which is nearly isotropic at lower energies. As a check, the differential cross section was used as a probability distribution to plot the average cosine of the scattering angle of the resulting leptons as a function of the initial neutrino energy and compared to the curve obtained
Figure 13: Fitted angular difference distributions for elastic electron collisions with 20 (left), 60 (middle) and 80 (right) MeV neutrinos.

Figure 14: Smearing plot of fitted $\cos \theta$ distributions for neutrino-electron elastic collisions at supernova neutrino energies.
Figure 15: Average cosine of lepton scattering angle for inverse beta decay interactions.

Figure 16: Distribution of dot products between the fitted lepton directions and the true neutrino directions for 20 MeV neutrinos.

in an earlier study [12] in Figure 15.

The distribution of dot products between the resulting fitted lepton direction and the true neutrino direction are shown in Figure 16, and the angular resolution across various initial neutrino energies is shown in Figure 17.

The form of the dot product distribution is significantly different when considering inverse beta decay reactions. In this case, the lepton has a nearly equal probability of being emitted in any direction, which is why the distribution shows no angular trend. The distribution does not appear flat because of the small number of events simulated; with better statistics, the plots would appear smoother because the error on each bin would be reduced. The angular resolution curve indicates that this reaction is not useful for extracting directional data at the low neutrino energies typically observed during supernova events, and only a slight
Figure 17: Plot of neutrino angular resolution for inverse beta decay interaction simulations created using SKDetSim.

Anisotropy appears at higher energies.

A smearing plot and matrix were also created to summarize the detector’s angular response to inverse beta decay interactions. The same binning scheme is used for this reaction; however, the values for distributions below 2.5 MeV were set to zero because the cross section is not defined for those energies. This is because the neutrino energy required for an inverse beta decay interaction has a minimum threshold. As before, the cos-theta distributions are fitted and used to produce Figure 18.
Figure 18: Smearing plot of fitted $\cos \theta$ distributions for inverse beta decay interactions at supernova neutrino energies.

4 Conclusions

This study characterized the angular response of a water Cherenkov detector with the configuration of Super-Kamiokande to two different neutrino reactions. The ability of BONSAI fitter to reconstruct low energy lepton events was first examined to determine the angular resolution without the added kinematical deviation from reactions and to compare distributions simulated using SKDetSim and WCSim. It was found that the WCSim does not yet realistically describe the angular distribution observed within the tank because the angular resolution curve does not match prior predictions as closely as SKDetSim. Further work and tuning must be done for WCSim simulations in order to match the physical behavior of Super-Kamiokande.

The angular response was studied for neutrino-electron elastic collisions and inverse beta decay interactions. Neutrino-electron elastic collisions are useful for pointing to supernovae because of the directionality of the reaction, while inverse beta decay interactions are important to understand because they occur much more frequently in water Cherenkov detectors and may degrade the pointing accuracy. Angular resolution curves showed that the forward pointing directionality of elastic electron collisions improves as the initial neutrino energy is increased, while inverse beta decay interactions are nearly isotropic at all energies.

The information of the detector angular response is summarized in the form of smearing matrices. These are extremely useful for studying supernova physics because they may be used with the SNOwGLoBES software package to simulate observed distributions of supernova burst neutrinos [14]. This will facilitate the study of core collapse physics and neutrino physics alike. Finally, providing an early warning to a supernova burst would be incredibly useful because photons emitted at the beginning moments of core collapse have been previously unobserved.
The best addition to this study would be an increase in the number of Monte Carlo statistics. Simulating a greater number of particles would reduce error and provide better results. Also, to better understand the pointing accuracy of water Cherenkov detectors, further studies should be done examining the angular response to charged current neutrino-oxygen interactions. This reaction has a slight reverse anisotropy, which may prove useful for pointing with very large statistics. In addition, neutrino reactions with oxygen comprise the dominant background to neutrino-electron collisions when inverse beta decay reactions are tagged. Another objective for future work should include exploring other neutrino detector configurations. In particular, understanding the response of liquid-argon detectors to supernova bursts would be helpful for flavor sensitivity studies.
5 Appendix A

The kinematics/trajectory may be described under special relativity by a simple application of relativistic kinematics as shown in Figure 18. The four-vectors of the particles may be approximated as the following:

\[
p_{\nu_i} = \begin{pmatrix} E_{\nu_i}^i \\ E_{\nu_i}^i \\ 0 \\ 0 \end{pmatrix}, \quad p_{e_i} = \begin{pmatrix} m_e \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad p_{\nu_f} = \begin{pmatrix} E_{\nu_f}^f \\ E_{\nu_f}^f \cos \theta_1 \\ E_{\nu_f}^f \sin \theta_1 \\ 0 \end{pmatrix}, \quad p_{e_f} = \begin{pmatrix} E_e + m_e \\ \sqrt{E_e^2 + 2m_eE_e \cos \theta_2} \\ \sqrt{E_e^2 + 2m_eE_e \sin \theta_2} \\ 0 \end{pmatrix}
\]

Momentum conservation gives:

\[
p_{\nu_i} + p_{e_i} = p_{\nu_f} + p_{e_f}
\]

This results in a system of four equations that give Equation 7 when solved.
References


