Lecture 21

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A. Goshaw Physics 346

The Standard Model's electroweak theory

- Examples of mass generation using the Higgs mechanism
  - Example 1: Real scalar fields generating fermion masses DONE
  - Example 2: Complex scalar fields and Goldstone bosons DONE
  - Example 3: Complex scalar fields generating boson masses DONE

- Complex scalar fields in iso-spin doublets: the Standard Model's electroweak theory (the Weinberg- Salam Model)
  - Introduction and overview
  - Working out the technical details
  - Generating W and Z boson masses
  - Generating fermion masses

Introduction to the Standard Model's Higgs mechanism

Introduction:
The Standard Model Higgs Mechanism

- What is the minimum complexity of the Higgs field sector required to generate the masses of the Standard Model's particles?
  1. The quarks and leptons
  2. The W^+ , W^- and Z^o weak vector bosons
  3. But also keep the photon and gluon masses = 0
- One real scalar field with multiple fermion couplings could handle 1.
- One complex field could do 1 and also create one massive vector boson
- However to accomplish 1, 2 and 3, two complex fields are required.
- One component of the field produces a scalar particle, the Higgs boson. The other 3 components would create mass-less Goldstone bosons, but are used to provide the mass to the W^+ , W^- and Z^o bosons using the process described in example 3.

Introduction:
The Standard Model Higgs Mechanism

- As in the previous examples, we will introduce the fundamental Higgs field ad hoc, and then show that it is sufficient to close the loop on the formulation of EWK theory.

- Of course this might not be the solution Nature has chosen, but it is the simplest way to get a self-consistent EWK theory. This solution is called the Weinberg-Salam Model.

- You will see that this procedure blends together elements of the examples 1, 2 and 3 previously discussed, so it should look familiar.

The Weinberg-Salam Model: the Higgs field input

- Let ϕ represent an SU(2) doublet of complex scalar fields

$$\phi = \begin{pmatrix} \phi^+(x) \\ \phi^o(x) \end{pmatrix} = \begin{pmatrix} [\phi_1(x) + i\,\phi_2(x)]/2^{1/2} \\ [\phi_3(x) + i\,\phi_4(x)]/2^{1/2} \end{pmatrix}$$

with quantum numbers:

	T	T_3	Q	$Y = 2(Q-T_3)$
$\phi^+(x)$	1/2	1/2	+1	1
$\phi^o(x)$	1/2	-1/2	0	1

The Weinberg-Salam Model: the philosophy

- Having postulated a Higgs field we now go back to lecture 16 where we developed an EWK theory with zero mass particles:

$$\mathcal{L}_o = -1/4\, B_{\mu\nu}B^{\mu\nu} - 1/4\, \mathbf{W}_{\mu\nu} \bullet \mathbf{W}^{\mu\nu} + \bar{\psi}\, i\, \gamma^\mu D_\mu \psi$$

with

$$D_\mu = \partial_\mu + i\, g\, [W_\mu^+ \; T_L^+ + W_\mu^- \; T_L^-] + i\, g_Z\, Z_\mu\, [T_{3L} - \sin^2\theta_W\, Q] + i\, e\, Q\, A_\mu$$

- This zero-mass Lagrangian $\mathcal{L}_o$ reproduced the dynamics of the weak interaction with $U_Y(1)\mathrm{x}SU_L(2)$ invariance but only if all masses = 0.

- Now use the Higgs Mechanism to repair this problem.

- In words, we introduce a scalar field, with a special potential, that interacts with the zero mass gauge fields (W, Z) and fermions and "gives them mass".

Details of the Standard Model Higgs mechanism for boson masses

The basic inputs and assumptions: 1. EWK zero-mass dynamics

1. Start with the zero-mass Lagrangian $\mathcal{L}_o$ that incorporates the $U_Y(1) \times SU_L(1)$ symmetry of electroweak dynamics.

$$\mathcal{L}_o = -1/4\, B_{\mu\nu}B^{\mu\nu} - 1/4\, \mathbf{W}_{\mu\nu} \bullet \mathbf{W}^{\mu\nu} + \bar{\psi}\, i\, \gamma^\mu D_\mu \psi$$

with
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$
$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - g\, \mathbf{W}_\mu \times \mathbf{W}_\nu$$
$$D_\mu = \partial_\mu + i\, g\, [W_\mu^+\ T^+_L + W_\mu^-\ T^-_L] + i\, g_Z\, Z_\mu\, [T_{3L} - \sin^2\theta_W\, Q] + i\, e\, Q\, A_\mu$$

and
$$T^+_L = (T_1 + i\, T_2)/2^{1/2}\ [1/2(1 - \gamma^5)]$$
$$T^-_L = (T_1 - i\, T_2)/2^{1/2}\ [1/2(1 - \gamma^5)]$$
$$T_{3L} = T_3\, [1/2(1 - \gamma^5)]$$

The basic inputs and assumptions: 1. EWK zero-mass dynamics

- The particle content includes the quarks and leptons in the weak isotopic spin doublets ψ and the gauge boson fields:

$$W_\mu^+ = (W_{1\,\mu} - i\,W_{2\,\mu})/2^{1/2}$$
$$W_\mu^- = (W_{1\,\mu} + i\,W_{2\,\mu})/2^{1/2}.$$

$$Z_\mu = W_{3\,\mu}\cos\theta_W - B_\mu \sin\theta_W$$

$$A_\mu = W_{3\,\mu}\sin\theta_W + B_\mu \cos\theta_W$$

- The free parameters are the electron charge $e = (4\pi\alpha)^{1/2}$ and the Weinberg angle θ_W.

$$g = e/\sin\theta_W$$
$$g_Z = e/(\sin\theta_W \cos\theta_W)$$

The basic inputs and assumptions:
2. The free field Higgs potential

2. Introduce a weak doublet of complex scalar fields

$$\phi = \begin{pmatrix} \phi^{+}(x) \\ \phi^{o}(x) \end{pmatrix} = \begin{pmatrix} [\phi_1(x) + i\,\phi_2(x)]/2^{1/2} \\ [\phi_3(x) + i\,\phi_4(x)]/2^{1/2} \end{pmatrix}$$

with a special form for the potential for the fields:

$$\mathcal{L}_\phi = (\partial_\mu \phi^\dagger)\ (\partial^\mu \phi) - [-\mu^2\ \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2]$$

where '†" means adjoint and ϕ is a column vector,
$\phi^\dagger$ a row vector and $\phi^\dagger \phi = (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)/2$

Two free parameters μ and λ are needed to describe the potential.

- This $\mathcal{L}_\phi$ is invariant under SU(2) ~ $(1 - i\,g\,\alpha(\mathbf{x}) \cdot \mathbf{T})$

The basic inputs and assumptions: 3. Introduce symmetry breaking

3. The minimum of the potential $V = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$ occurs at $\phi^\dagger \phi = \mu^2/(2\lambda)$ or:

$$\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \mu^2/\lambda$$

Break the symmetry by choosing one point on the minimum potential surface:

$$\phi_1 = \phi_2 = \phi_4 = 0 \text{ and } \phi_3 = (\mu^2/\lambda)^{1/2}$$

Let the symbol v = the vacuum expectation value = $(\mu^2/\lambda)^{1/2}$.
As before this does not introduce any additional free parameters.

The basic inputs and assumptions: 3. Introduce symmetry breaking

- Use the freedom to choose the functions $\alpha_i(x)$ with i = 1,2,3 to eliminate three of the scalar fields ϕ_1 , ϕ_2 qnd ϕ_4 (see example 3).

- Expand the one remaining real field ϕ_3 about the point chosen on the potential minimum surface $v = (\mu^2/\lambda)^{1/2}$

$$\phi(x) = 1/2^{1/2} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

The basic inputs and assumptions: 4. Construct the EWK Lagrangian

4. Add the complex scalar field doublet described above by $\mathcal{L}_\phi$ to the zero-mass electroweak Lagrangian $\mathcal{L}_o$:

$$\mathcal{L}_{EWK} = -1/4\, B_{\mu\nu}B^{\mu\nu} \;\; -1/4\, \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu} \;+\; \overline{\psi}\; i\gamma^\mu D_\mu \psi$$

$$+\, (D_\mu\, \phi)^\dagger (D^\mu\, \phi) - \;[-\mu^2\, \phi^\dagger \phi \,+\, \lambda\, (\phi^\dagger \phi)^2]$$

Here we have introduced an interaction between the scalar field ϕ and the gauge bosons by replacing ∂_μ in $\mathcal{L}_\phi$ by D_μ = the covariant derivative (see page 8).
Call this term $\mathcal{L}_G$.

- $\mathcal{L}_G$ introduces couplings of the gauge fields to the Higgs field H and generates mass terms for the $W^\pm$ and Z^o bosons.

Calculation of boson masses

- Now show that the *gauge* boson masses are generated from:

$$\mathcal{L}_G = (D_\mu \phi)^\dagger (D^\mu \phi) - [-\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2]$$

with
$$D_\mu = \partial_\mu + i g [W_\mu^+ T^+ + W_\mu^- T^-] + i g_Z Z_\mu [T_3 - \sin^2\theta_W Q] + i e Q A_\mu$$

(I have dropped the "L" subscript on the T's since here the T's do not act upon any fermion spinor states)

Calculation of boson masses

- Substitute the explicit expressions for ϕ and the T matrices:

$$\phi(x) = 1/2^{1/2} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

and the 2x2 matrices T^+ , T^- and T_3 are:

$$2^{1/2}\, T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad 2^{1/2}\, T^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad 2T_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Calculation of boson masses

- Substitute the expressions for $\phi(x)$ into $\mathcal{L}_G$.
- After a page of arithmetic you can easily show that:

$$\mathcal{L}_G = 1/2\,(\partial_\mu H)(\partial^\mu H)$$
$$+\,1/4\,g^2 W_\mu^+ W^{\mu -} (v+H)^2 + 1/8\,g_Z^2 Z_\mu Z^\mu (v+H)^2$$
$$-\,[\,-\mu^2/2 + \lambda/4(v+H)^2](v+H)^2$$

- See lecture 17 notes to identify the following boson mass terms:
  - for the W boson: $+\,m_W^2 W_\mu^+ W^{\mu -}$
  - for the Z boson: $+1/2\,m_Z^2 Z_\mu Z^\mu$
  - for the H boson: $-1/2\,m_H^2 H^2$
- By identifying the mass terms in $\mathcal{L}_G$ the masses for the $W^\pm$, Z^o and H^o bosons can be identified.

$m_W = gv/2$ $\quad$ $m_Z = g_Z v/2$ $\quad$ $m_H = (2\mu^2)^{1/2}$

with $\mu^2 = v^2 \lambda$

You should show this.

Calculation of boson couplings

- The model also predicts cubic and quartic couplings of the Higgs field with itself and with the weak gauge bosons:

  $\sim \lambda v\, H^3 + \lambda/4\, H^4 + (1/4\, g^2\, W_\mu^+ W^{\mu\,-} + 1/8\, g_Z^2\, Z_\mu Z^\mu)(H^2 + 2v\, H)$

  - The self couplings of the Higgs boson depends on λ
  - The couplings of the Higgs boson to the weak bosons depends only on the gauge couplings g and g_Z .

- *All this structure is completely fixed by this Standard Model of the electroweak interaction.*

Summary of the SM EWK predictions

- Now let's collect together all the predictions.

1. Calculate $\sin^2\theta_W$

From $m_W = gv/2$ and $m_Z = g_Z v/2$

and using $g = e/\sin\theta_W$ and $g_Z = e/[\sin\theta_W \cos\theta_W]$

$$\Rightarrow \quad \boxed{\sin^2\theta_W = [1 - (m_W/m_Z)^2]}$$

With $m_W = 80.425\ \mathrm{GeV}/c^2$ and $m_Z = 91.188\ \mathrm{GeV}/c^2$ $\sin^2\theta_W = 0.222$

- The Weinberg angle is no longer a free parameter but fixed in value by the W and Z masses.

Summary of the SM EWK predictions

2. Calculate the Higgs boson vacuum energy v

$$v = 2\, m_W/g = 2\, m_W/[e\,/\sin\theta_W]$$

Therefore: $v = 2\, m_W\,[1 - (m_W/m_Z)^2]^{1/2}/(4\pi\alpha)^{1/2}$

With $m_W = 80.425$ GeV/c^2 , $m_Z = 91.188$ GeV/c^2 and $\alpha = 1/128$

$$v = 246 \text{ GeV}$$

- The Higgs vacuum expectation value is no longer a free parameter but fixed in value by other Standard Model parameters.
- This large vacuum energy expectation value of the Higgs potential is odd. It does not detract from the self-consistency of the theory, but if taken at face value would imply a large cosomological energy density in the universe that is not observed. This is a hint that at a deeper level the SM is incomplete.

Summary of the SM EWK predictions

3. The Higgs boson mass m_H

The Higgs boson mass is: $m_H = (2\mu^2)^{1/2}$

but since $\mu^2 = v^2 \lambda$, although v is known λ is a free parameter.

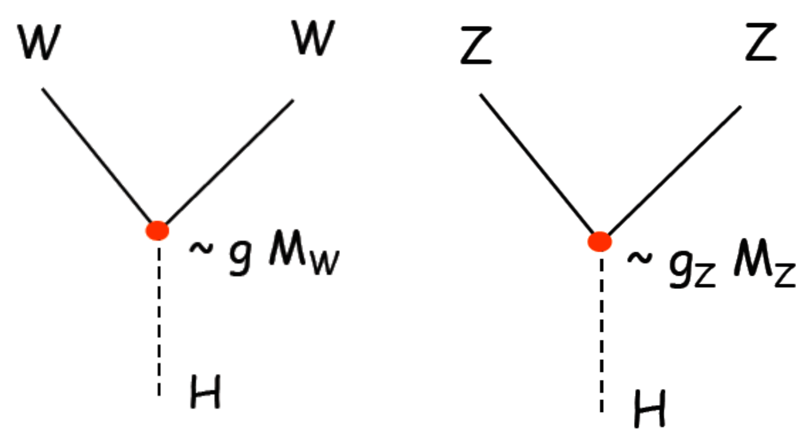
- Therefore the mass of the Higgs boson is a free parameter, and must be measured.

- If the Higgs boson is discovered and its mass measured, then the parameters describing the Higgs potential (μ and λ), are fixed and the model is completely predictive.

Summary of the SM EWK predictions

4. Self couplings of the gauge bosons to the Higgs boson.

All the couplings between the gauge bosons and the Higgs boson are fixed in terms of $e = (4\pi\alpha)^{1/2}$, λ and the gauge boson masses.



$g = e/\sin\theta_W$

$g_Z = e/(\sin\theta_W \cos\theta_W)$

$\sin^2\theta_W = [1 - (M_W/M_Z)^2]$

Summary of the SM EWK predictions

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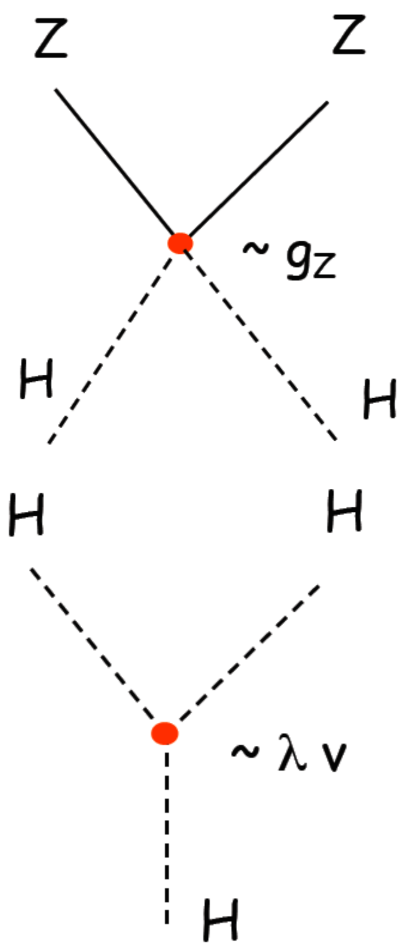

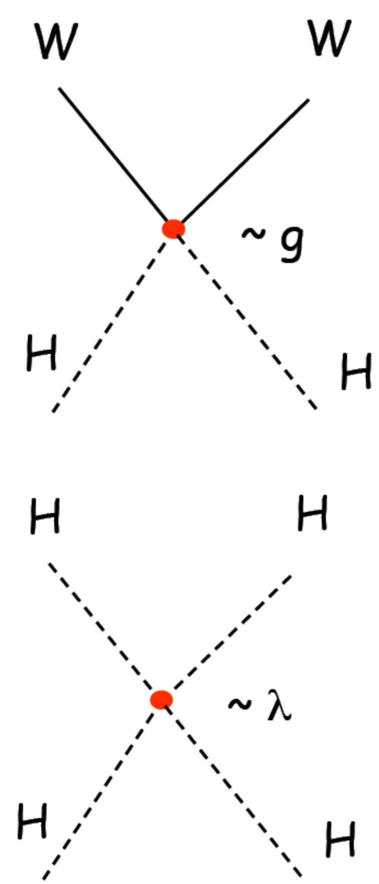

More W-Z-H couplings

Next Lecture
Standard Model Higgs mechanism for fermion masses